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GEOTECHNICAL ENGINEERING – I (Subject Code: 06CV54)
UNIT 4: FLOW OF WATER THROUGH SOILS

Contents:

Darcy's law- assumption and validity, coefficient of permeability and its determination (laboratory and field), factors affecting permeability, permeability of stratified soils, Seepage velocity, Superficial velocity and coefficient of percolation, effective stress concept-total pressure and effective stress, quick sand phenomena, Capillary Phenomena.

4.1 Introduction:

Water strongly affects engineering behaviour of most kind of soils and water is an important factor in most geotechnical engineering problems. Hence it is essential to understand basic principles of flow of water through soil medium. Flow of water take place through interconnected pores between soil particles is considered in one direction.

Objectives of this chapter are to understand basic principles of one dimensional flow through soil media. This understanding has application in the problems involving seepage flow through soil media and around impermeable boundaries which are frequently encountered in the design of engineering structures. To understand the concepts involved in this particular subject, student is advised to review basic terminologies of fluid mechanics course [06 CV 35 – Fluid Mechanics]. This is essential because, as water flows through soil medium from a higher energy to a lower energy the concerned modeling is governed by the principles of fluid mechanics.

4.2 Permeability

Flow of water in soil media takes place through void spaces which are apparently interconnected. Water can flow through the densest of natural soils. Water does not flow in a straight line but in a winding path (tortuous path) as shown in Figure 4.1. However, in soil mechanics, flow is considered to be along a straight line at an effective velocity. The velocity of drop of water at any point along its flow path depends on the size of the pore and its position inside the pore.

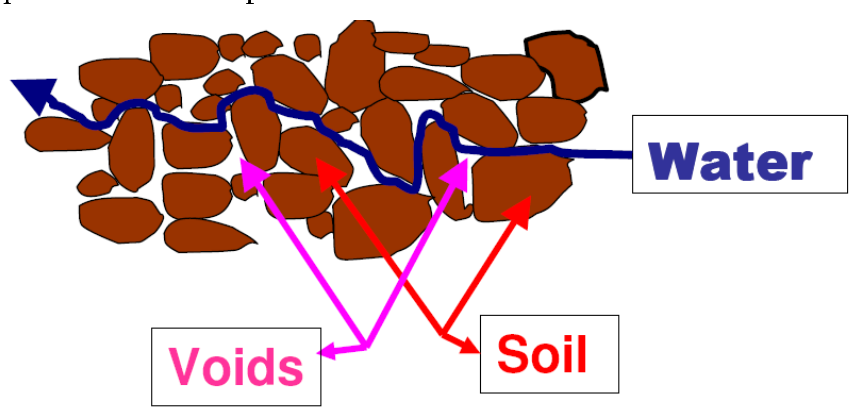

Figure 4.1: Water flows in a winding path through soil media

Since water can flow through the pore spaces in the soil hence soil medium is considered to be *permeable*. Thus, the property of a porous medium such as soil by virtue of which water can flow through it is called its ***permeability***. In other words, the ease with which water can flow through a soil mass is termed as permeability.

4.3 Darcy's law

In 1856, modern studies of groundwater began when French scientist and engineer, *H.P.G. Darcy (1803-1858)* was commissioned to develop a water-purification system for the city of Dijon, France. He constructed the first experimental apparatus to study the flow characteristics of water through the soil medium. From his experiments, he derived the equation that governs the laminar (non-turbulent) flow of fluids in homogeneous porous media which became to be known as Darcy's law.

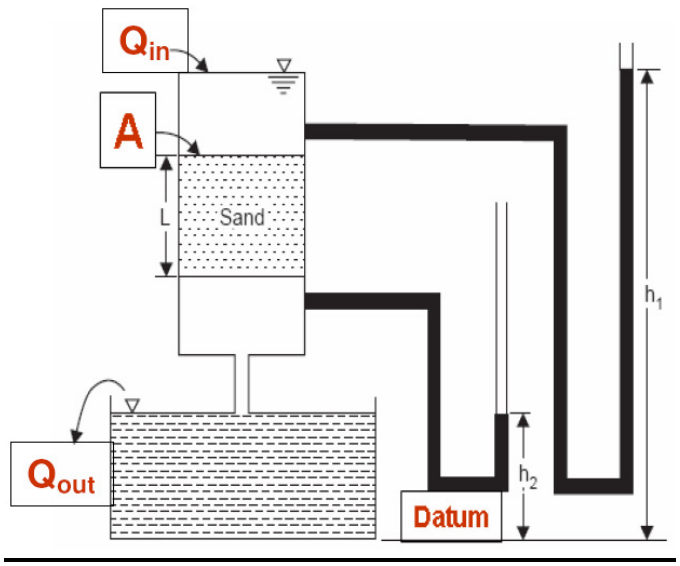

Figure 4.2: Schematic diagram depicting Darcy's experiment

The schematic diagram representing Darcy's experiment is depicted in Figure 4.2. By measuring the value of the rate of flow, Q for various values of the length of the sample, L, and pressure of water at top and bottom the sample, h_1 and h_2, Darcy found that Q is proportional to $(h_1 - h_2)/L$ or the hydraulic gradient, i, that is,

$$Q = k\frac{h_1 - h_2}{L}A = k\frac{\Delta h}{L}A$$

$$Q = kiA$$

The loss of head of Δh units is affected as the water flows from h_1 to h_2. The hydraulic gradient defined as loss of head per unit length of flow may be expressed as,

$$i = \frac{\Delta h}{L}$$

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k is coefficient of permeability or hydraulic conductivity with units of velocity, such as mm/sec or m/sec. Thus the theory of seepage flow in porous media is based on a generalization of Darcy's Law which is stated as, "***Velocity of flow in porous soil media is proportional to the hydraulic gradient***" where, flow is assumed to be laminar. That is,

$$v = k\ i$$

Where, ***k*** is coefficient of permeability, v is velocity of flow and i is the hydraulic gradient. Typical values of coefficient of permeability for various soils are as follows,

Soil type	*Coefficient of permeability (mm/s)*
Coarse	$10 - 10^3$
Fine gravel, coarse, and medium sand	$10^{-2} - 10$
Fine sand, loose silt	10^{-4} – 10–2
Dense silt, clayey silt	$10^{-5} - 10^{-4}$
Silty clay, clay	$10^{-8} - 10^{-5}$

Assumptions made defining Darcy' law.

- The flow is laminar that is, flow of fluids is described as laminar if a fluid particles flow follows a definite path and does not cross the path of other particles.
- Water & soil are incompressible that is, continuity equation is assumed to be valid
- The soil is saturated
- The flow is steady state that is, flow condition do not change with time.

4.4 Seepage velocity and Superficial velocity

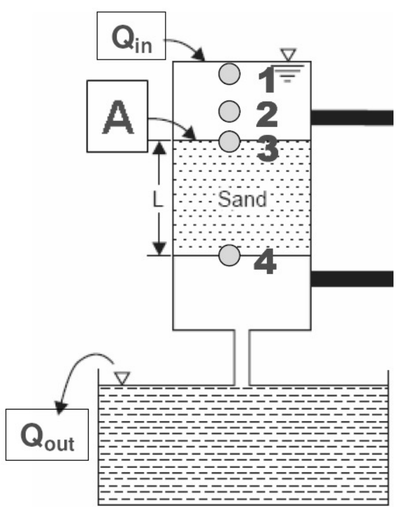


Figure 4.3: Seepage velocity and superficial velocity

Consider flow of water through soil medium of length *L* and cross sectional area *A* as shown in Figure 4.3. If *v* is the velocity of downward movement of a drop of water from positions 1 to 2 then velocity is equal to *ki,* therefore *k* can be interpreted as the 'approach velocity' or 'superficial velocity' for unit hydraulic gradient, i.e.,. $k = \frac{v}{(i=1)} = k$

Drop of water flows from positions 3 to 4 at faster rate than it does from positions 1 to 2 because the average area of flow channel through the soil is smaller. The actual velocity of water flowing through the voids is termed as seepage velocity, v_s.

By the principle of continuity, the velocity of approach $\boldsymbol{v}$, may be related to the seepage velocity or average effective velocity of flow, $\boldsymbol{v_s}$ by equating Q_{in} and Q_{out} as follows,

$$Q = vA = v_s A_v$$

$$v_s = v\frac{A}{A_v} = v\frac{AL}{A_v L} = v\frac{V}{V_v}$$

$$v_s = \frac{v}{n} = \frac{1+e}{e}v = \frac{ki}{n} = \frac{1+e}{e}ki$$

Where, A_v = Area of pores, V = total volume of soil, V_v = volume of voids, e = voids ratio and n = porosity. Thus seepage velocity is the superficial velocity divided by the porosity. Above equation indicates that the seepage velocity is also proportional to the hydraulic gradient.

$$v_s = \frac{k}{n}i = \frac{(1+e)k}{e}i = k_p i,$$

That is, $\boldsymbol{v_s \propto k_p}$, where $\boldsymbol{k_p}$ is called coefficient of percolation given by,

$$k_p = \frac{k}{n} = \frac{(1+e)k}{e}$$

Thus, coefficient of percolation is defined as the ratio of coefficient of permeability to porosity.

4.5 Factors affecting permeability

The coefficient of permeability as used by geotechnical engineer is the approach or superficial velocity of the permeant flowing through soil medium under unit hydraulic gradient hence it depends on the characteristics of permeant, as well as those of the soil. Considering the flow through a porous medium as similar to a flow through a bundle of straight capillary tubes, the relationship showing the dependency of soil permeability on various characteristic parameters of soil and permeant was developed by Taylor as given below,

$$k = D^2 \frac{\rho}{\mu} \frac{e^3}{(1+e)} C$$

This equation reflects the influence of permeant and the soil characteristics on permeability. In the above equation D is the effective diameter of the soil particles, ρ is the unit weight of fluid, μ is the viscosity of fluid and C is the shape factor. Therefore, with the help of above equation, factors affecting permeability can be listed as follows,

Permeant fluid properties

i. **Viscosity of the permeant**
ii. **Density and concentration of the permeant.**

Soil characteristics

i. **Grain-size**
   - Shape and size of the soil particles.
   - Permeability varies with the square of particle diameter.
   - Smaller the grain-size the smaller the voids and thus the lower the permeability.
   - A relationship between permeability and grain-size is more appropriate in case of sands and silts.
   - Allen Hazen proposed the following empirical equation, $k(cm/s) = C\, D_{10}^2$, C is a constant that varies from 1.0 to 1.5 and D_{10} is the effective size, in mm

ii. **Void ratio**
   - Increase in the porosity leads to an increase in the permeability.
   - It causes an increase in the percentage of cross-sectional area available for flow.

iii. **Composition**
   - The influence of soil composition on permeability is generally insignificant in the case of gravels, sands, and silts, unless mica and organic matter are present.
   - Soil composition has major influence in the case of clays.
   - Permeability depends on the thickness of water held to the soil particles, which is a function of the cation exchange capacity.

iv. **Soil structural**
   - Fine-grained soils with a flocculated structure have a higher coefficient of permeability than those with a dispersed structure.
   - Remoulding of a natural soil reduces the permeability
   - Permeability parallel to stratification is much more than that perpendicular to stratification

v. **Degree of saturation**
   - Higher the degree of saturation, higher is the permeability.
   - In the case of certain sands the permeability may increase three-fold when the degree of saturation increases from 80% to 100%.

vi. **Presence of entrapped air and other foreign matter.**
   - Entrapped air reduces the permeability of a soil.
   - Organic foreign matter may choke flow channels thus decreasing the permeability

Hence, it is important to simulate field conditions in order make realistic estimate of the permeability of a natural soil deposit, particularly when the aim is to determine field permeability in the laboratory.

4.6 Validity or limitations of Darcy's law

Darcy's law given by $\boldsymbol{v = k\ i}$ is true for laminar flow through the void spaces. A criterion for investigating the range can be furnished by the Reynolds number. For flow through soils, Reynolds number R_n can be given by the relation,

$$R_n = \frac{vD\rho}{\mu g}$$

Where, v=discharge (superficial) velocity in cm/s, D = average diameter of the soil particle in cm, ρ= density of the fluid in g/cm^3, μ= coefficient of viscosity in g/cm·s, g = acceleration due to gravity, cm/s^2

For laminar flow conditions in soils, experimental results show that,

$$R_n = \frac{vD\rho}{\mu g} \leq 1$$

with coarse sand, assuming D = 0.47 mm and $k \approx 100D^2$ = 100(0.047)2 = 0.2209 cm/s.
Assuming i = 1, then $v = ki$ = 0.2209 cm/s.
Also, water ≈ 1g/cm^3, and (μ at 20^{0}C) g = 10–5 (980) g/cm·s. Gives, $\boldsymbol{R_n = 1.059} \approx 1$

From the above calculations, we can conclude that, for flow of water through all types of soil (sand, silt, and clay), the flow is laminar and Darcy's law is valid. With coarse sands, gravels, and boulders, turbulent flow of water can be expected

4.7 Measurement of coefficient of permeability – Laboratory tests

There are two types of tests available for determining coefficient permeability in the laboratory, namely,

i. **Constant-head permeability test -** suitable for *coarse grained soils*,
ii. **Falling or Variable -head permeability test -** suitable for *fine grained soils*,

Constant-head permeability test

The constant-head test is suitable for more permeable coarse grained soils. The basic laboratory test arrangement is shown in Figure 4.4. The soil specimen is placed inside a cylindrical mold, and the constant-head loss h of water flowing through the soil is maintained by adjusting the supply. The outflow water is collected in a measuring cylinder, and the duration of the collection period is noted. From Darcy's law, the total quantity of flow Q in time t can be given by

$$Q = qt = kiAt$$

$$\therefore\ k = \frac{Q}{At}\frac{L}{h}$$

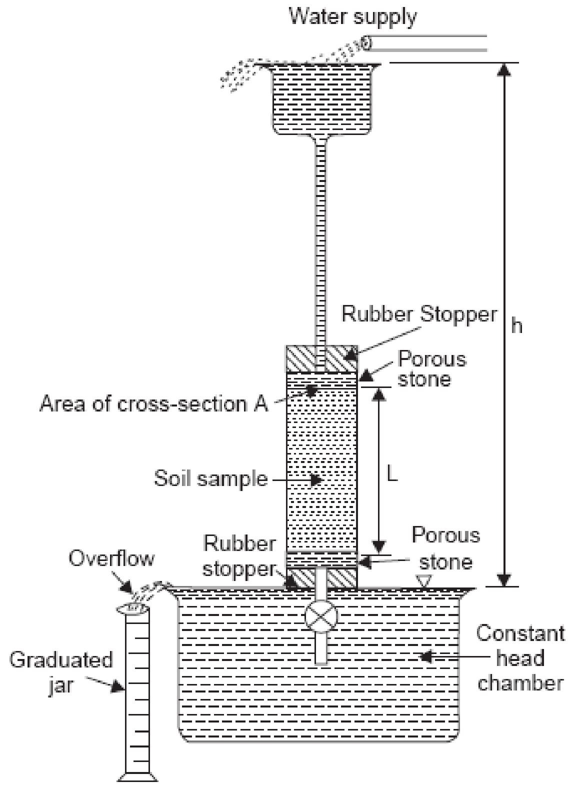


Figure 4.4: Constant-head permeability test

Where, A is the area of cross-section of the specimen and $i = h/L$, where L is the length of the specimen.

Falling or Variable -head permeability test

The falling-head permeability test is more suitable for fine-grained soils. Figure 4.5 shows the general laboratory arrangement for the test. The soil specimen is placed inside a tube, and a standpipe is attached to the top of the specimen. Water from the standpipe is let to flow through the soil specimen. The initial head difference h_1 at time $t = t_1$ is recorded, and water is allowed to flow through the soil such that the final head difference at time $t = t_2$ is h_2. The rate of flow through the soil is

Velocity of fall of water level; $v = -\frac{dh}{dt}$

Flow of water into the sample; $q_{in} = -a\frac{dh}{dt}$

From Darcy's law flow out of the sample; $q_{out} = kiA = k\frac{h}{L}A$

For continuity; $q_{in} = q_{out}$

$$\Rightarrow \quad -a\frac{dh}{dt} = k\frac{h}{L}A$$

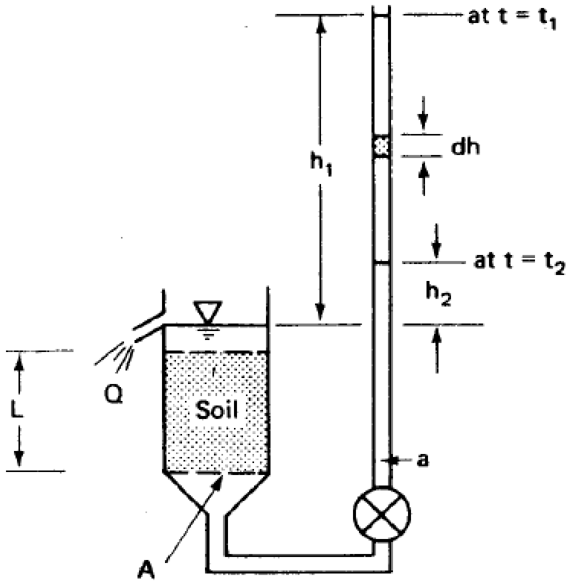


Figure 4.5: Falling or Variable -head permeability test

Seperating the variables and integrating over the limits,

$$\Rightarrow \quad a\int_{h_2}^{h_1} \frac{dh}{h} = k\frac{A}{L}\int_{t_2}^{t_1} dt;$$

$$\Rightarrow \quad a\ln\left(\frac{h_1}{h_2}\right) = k\frac{A}{L}(t_1 - t_2)$$

$$\Rightarrow \quad k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

where $t = t_1 - t_2$, interms of ($\log_{10}$)

$$\Rightarrow k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$

Thus measuring h_1 and h_2 during time t, k can be computed.

Note: For constant t, at two instances, let water level fall from h_1 *to* h_2 and h_2 *to* h_3, then, $h_2 = \sqrt{h_1 h_3}$

4.8 Measurement of coefficient of permeability – Field tests

The coefficient of permeability of the permeable layer can be determined by pumping from a well at a constant rate and observing the steady-state water table in nearby observation wells. The steady-state is established when the water levels in the test well and the observation wells become constant. When water is pumped out from the well, the aquifer gets depleted of water, and the water table is lowered resulting in a circular depression in the phreatic surface. This is referred to as the ‘Drawdown curve’ or ‘Cone of depression’. The analysis of flow towards such a well was given by Dupuit (1863).

Assumptions

1. The aquifer is homogeneous.
2. Darcy's law is valid.
3. The flow is horizontal.
4. The well penetrates the entire thickness of the aquifer.
5. Natural groundwater regime remains constant with time.
6. Dupuit's theory is valid that is, $i = dz/dr$

Case 1: Unconfined Aquifer

Let ***r*** and ***z*** be the radial distance and height above the impervious boundary at any point on the drawdown curve as shown in Figure 4.6. At steady state, the rate of discharge due to pumping can be expressed as, $q = kiA$

Hydraulic gradient at any point is given by Dupuit's theory, $i = \dfrac{dz}{dr}$

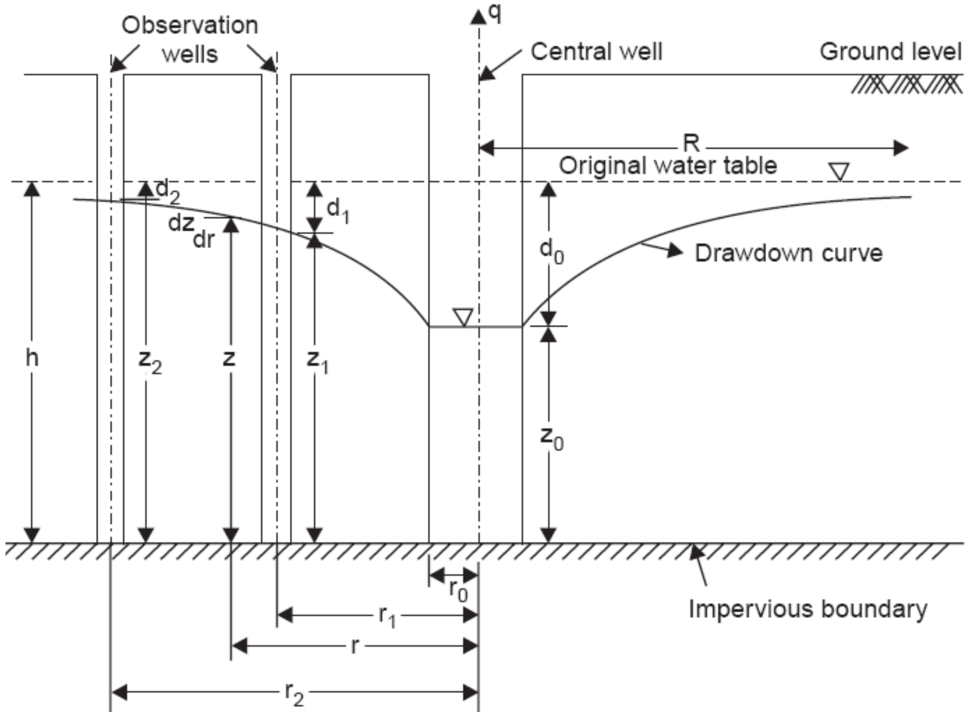


Figure 4.6: Field permeability test – Pumping out test in an unconfined aquifer

$$\therefore q = k\frac{dz}{dr}2\pi rz \qquad (\because A = 2\pi rz)$$

$$\frac{dr}{r} = \frac{2\pi k}{q}(z.dz), \qquad \text{integrating both sides } \int_{r_1}^{r_2}\frac{dr}{r} = \frac{2\pi k}{q}\int_{z_1}^{z_2} z.dz$$

$$\Rightarrow k = \frac{2.303q}{\pi(z_2^2 - z_1^2)}\left[\log_{10}\frac{r_2}{r_1}\right]$$

Note: $z_1 = (h - d_1)$ & $z2 = (h - d_2)$

$$\therefore k = \frac{2.303q}{\pi[(d_1 - d_2)(2h - d_1 - d_2)]}\left[\log_{10}\frac{r_2}{r_1}\right]$$

If the values of r_1, r_2, z_1, z_2, and q are known from field measurements, the coefficient of permeability can be calculated using the above relationship for *k*.

Case 2: Confined Aquifer

Let ***r*** and ***z*** be the radial distance and height above the impervious boundary at any point on the drawdown curve as shown in Figure 4.7. At steady state, the rate of discharge due to pumping can be expressed as, $q = kiA$

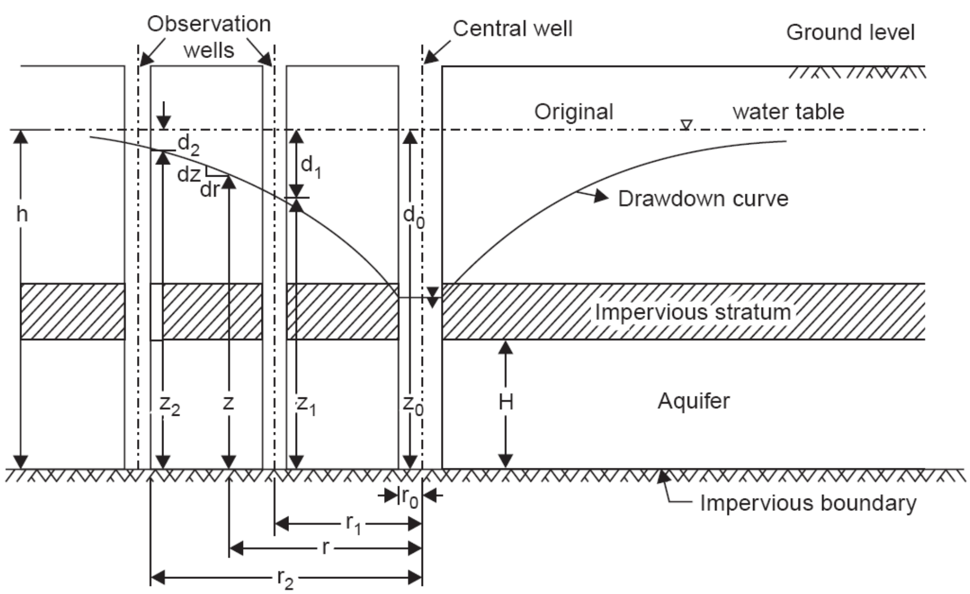


Figure 4.7: Field permeability test – Pumping out test in confined aquifer

$$\therefore q = k\frac{dz}{dr}2\pi rH \qquad (\because A = 2\pi rH)$$

H is depth of confined aquifer

$kdz = \frac{q}{2\pi H}\frac{dr}{r}$ integrating both sides, $k\int_{z_1}^{z_2} dz = \frac{q}{2\pi H}\int_{r_1}^{r_2}\frac{dr}{r}$

$$\Rightarrow k[z]_{z_1}^{z_2} = \frac{q}{2\pi H}\left[\log_e r\right]_{r_1}^{r_2}$$

$$\Rightarrow k = \frac{2.303q}{2\pi H(z_2 - z_1)}\left(\log_{10}\frac{r_2}{r_1}\right); \qquad \Rightarrow k = \frac{q}{2.7283\times H(z_2 - z_1)}\left(\log_{10}\frac{r_2}{r_1}\right)$$

Note: $z_1 = (h - d_1)$ & $z2 = (h - d_2)$

$$\therefore k = \frac{q}{2.7283\times H(d_1 - d_1)}\left(\log_{10}\frac{r_2}{r_1}\right)$$

If the values of r_1, r_2, z_1, z_2, and q are known from field measurements, the coefficient of permeability can be calculated using the above relationship for k.

4.9 Permeability of stratified soils

Case 1: Flow perpendicular to the layers.

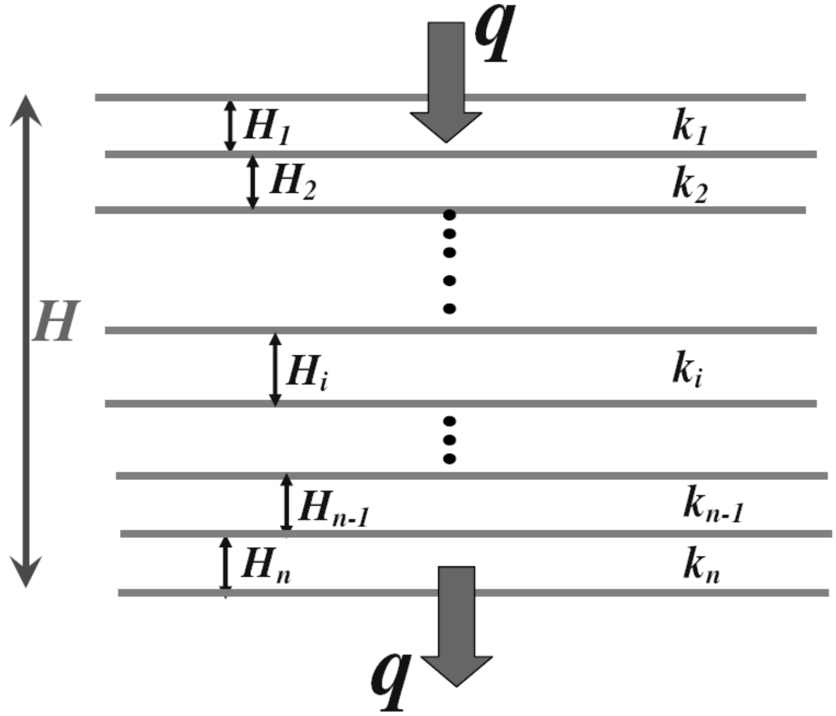


Figure 4.7: Effective Permeability of stratified soils - perpendicular to the layers.

If $\Delta H_1,\ \Delta H_2,\ \bullet\bullet\bullet\Delta H_i,\ \bullet\bullet\bullet\Delta H_{n-1}\ \Delta H_n$

are head lost in each of the corresponding layers

Then the total head lost ΔH is given by,

$$\Delta H = \Delta H_1 + \Delta H_2 + \bullet\bullet\bullet + \Delta H_i \bullet\bullet\bullet + \Delta H_{n-1} + \Delta H_n$$

Hydraulic gradients in each of these layers are,

$$i_1 = \frac{\Delta H_1}{H_1},\ i_2 = \frac{\Delta H_2}{H_2}, \bullet\bullet\bullet i_i = \frac{\Delta H_i}{H_i}, \bullet\bullet\bullet i_{n-1} = \frac{\Delta H_{n-1}}{H_{n-1}},\ i_n = \frac{\Delta H_n}{H_n}$$

Since discharge q is same in all the layers, the velocity is the same in all layers

ie., $v_1 = v_2 = \bullet\bullet\bullet = v_i = \bullet\bullet\bullet = v_{n-1} = v_n$

Let k_v be the average permeability perpendicular (vertical) to the bedding planes

then, $v = k_v i = k_1 i_1 = k_2 i_2 = \bullet\bullet\bullet = k_i i_i = \bullet\bullet\bullet = k_{n-1} i_{n-1} = k_n i_n$

ie.,

$$v = k_v \frac{\Delta H}{H} = k_1 \frac{\Delta H_1}{H_1} = k_2 \frac{\Delta H_2}{H_2} = \bullet\bullet\bullet = k_i \frac{\Delta H_i}{H_i} = \bullet\bullet = k_{n-1} \frac{\Delta H_{n-1}}{H_{n-1}} = k_n \frac{\Delta H_n}{H_n}$$

$$\therefore \Delta H = \frac{vH}{k_v},\ \Delta H_1 = \frac{vH_1}{k_1},\ \Delta H_2 = \frac{vH_2}{k_2}, \bullet\bullet\bullet, \Delta H_i = \frac{vH_i}{k_i} \bullet\bullet, \Delta H_{n-1} = \frac{vH_{n-1}}{k_{n-1}}, \Delta H_n = \frac{vH_n}{k_n}$$

Substituting in the equation, $\Delta H = \Delta H_1 + \Delta H_2 + \bullet\bullet\bullet + \Delta H_i + \bullet\bullet + \Delta H_{n-1} + \Delta H_n$

$$\Rightarrow \frac{vH}{k_v} = \frac{vH_1}{k_1} + \frac{vH_2}{k_2} + \bullet\bullet\bullet + \frac{vH_i}{k_i} + \bullet\bullet\bullet + \frac{vH_{n-1}}{k_{n-1}} + \frac{vH_n}{k_n}$$

$\therefore$ Effective permeability for flow in the direction perpendicular to layers is,

$$\Rightarrow k_v = \frac{H}{\left(\frac{H_1}{k_1} + \frac{H_2}{k_2} + \ldots + \frac{H_i}{k_i} + \ldots + \frac{H_{n-1}}{k_{n-1}} + \frac{H_n}{k_n}\right)}$$

Case 2: Flow parallel to the layers.

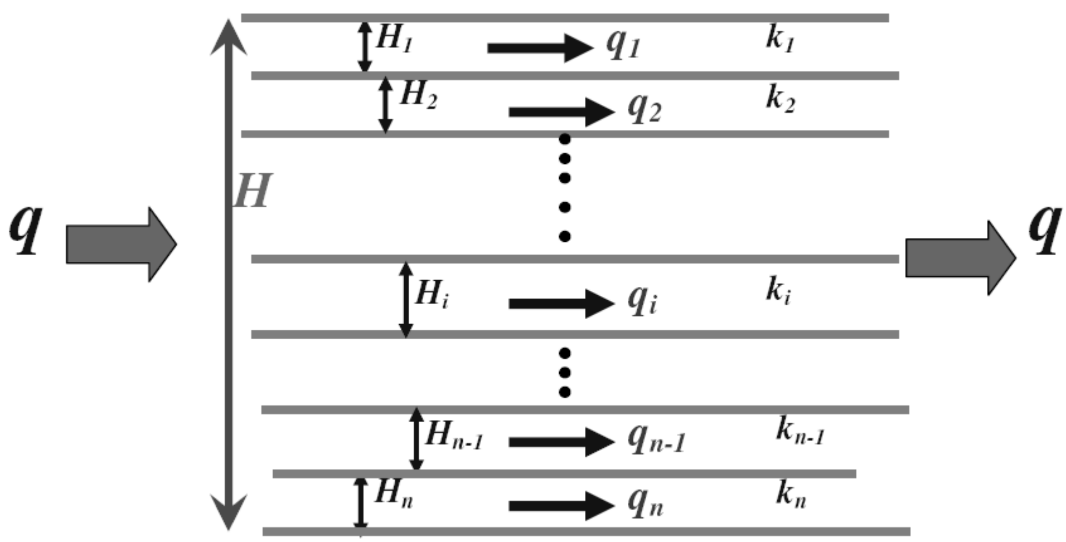


Figure 4.8: Effective Permeability of stratified soils – parallel to the layers

If $q_1, \; q_2, \; \bullet\bullet\bullet q_i, \; \bullet\bullet\bullet q_{n-1} \; q_n$

are rate of flow in each of the corresponding layers

Then the total rate of flow q is given by,

$q = q_1 + q_2 + \bullet\bullet\bullet + q_i \bullet\bullet\bullet + q_{n-1} + q_n$

Hydraulic gradients in each of these layers are same.

$i_1 = i_2 = \bullet\bullet\bullet i_i = \bullet\bullet\bullet = i_{n-1} = i_n = i$

Since discharge, $q = q_1 + q_2 + \bullet\bullet\bullet + q_i \bullet\bullet\bullet + q_{n-1} + q_n$

ie., $q = k_1 i H_1 + k_2 i H_2 + \bullet\bullet\bullet + k_i i H_i + \bullet\bullet\bullet + k_{n-1} i H_{n-1} + k_n i H_n$

Let k_h be the average permeability parallel (Horizontal)

to the bedding planes then, Note that for flow in the horizontal direction

(which is the direction of stratification of the soil layers),

the hydraulic gradient is the same for all layers.

$k_h i H = k_1 i H_1 + k_2 i H_2 + \bullet\bullet\bullet + k_i i H_i + \bullet\bullet\bullet + k_{n-1} i H_{n-1} + k_n i H_n$

$$\therefore \; k_h = \left[\frac{k_1 H_1 + k_2 H_2 + \ldots + k_i H_i + \ldots + k_{n-1} H_{n-1} + k_n H_n}{H} \right]$$

k_v and k_h are effective are equivalent permeability coefficients for flow in parallel and perpendicular directions to the bedding Planes respectively **[Note: $k_v < k_h$].**

4.10 Total Stress, Effective Stress and Pore Pressure

- External loading increases the total stress at every point in a saturated soil above its initial value.
- The magnitude of this increase depends mostly on the location of the point
- The pressure transmitted through grain to grain at the contact points through a soil mass is termed as inter-granular or effective pressure.
- It is known as effective pressure since this pressure is responsible for the decrease in the void ratio or increase in the frictional resistance of a soil mass.
- If the pores of a soil mass are filled with water and if a pressure induced into the pore water, tries to separate the grains, this pressure is termed as pore water pressure or neutral stress. It is the same in all directions

The total stress, either due to self-weight of the soil or due to external applied forces or due to both, at any point inside a soil mass is resisted by the soil grains as also by water present in the pores or void spaces in the case of a saturated soil.

Total stress = Effective stress + Pore water Pressure (Neutral stress)

$$\sigma = \sigma' + u$$

If γ_b and γ_w are soil bulk unit weight and unit weight of water respectively, then at any depth, z, Effective, total and neutral stresses relationship may be expressed as

$$\sigma' = \sigma - u = \gamma_b z - \gamma_w z$$

$$\sigma' = z(\gamma_b - \gamma_w) = \gamma' z$$

where, $u = \gamma_w z$.

Usually the unit weight of soil varies with depth. Soil becomes denser with depth owing to the compression caused by the geostatic stresses. If the unit weight of soil varies continuously with depth, the vertical stress at any point, σ_v can be evaluated by means of the integral:

$$\sigma_v = \int_0^z \gamma . dz$$

If the soil is stratified, with different unit weights for each stratum, σ_v may be computed conveniently by summation:

$$\sigma_v = \sum_{i=1}^{n} \gamma .(\Delta z)_i$$

4.11 Upward flow – Quick sand condition and Critical hydraulic gradient

Consider a case of water flowing under a hydraulic head $\boldsymbol{x}$ through a soil column of height $\boldsymbol{H}$ as shown in the Figure 4.9.

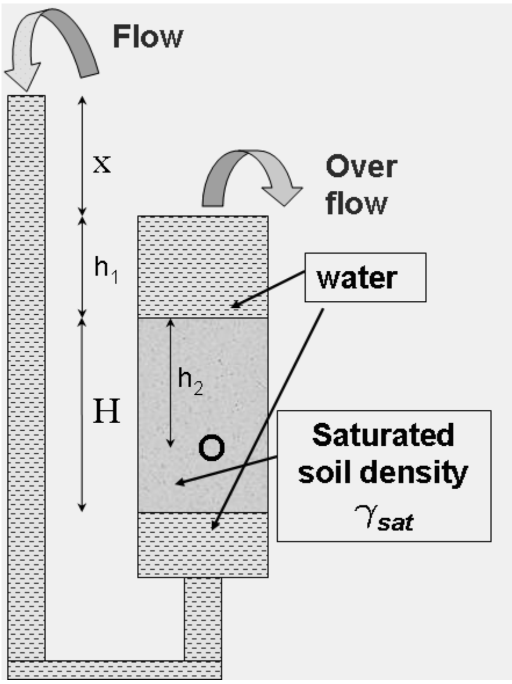


Figure 4.9: Computation of critical hydraulic gradient at point O.

The state of stress at point O situated at a depth of h_2 from the top of soil column may be computed as follows,

Vertical stress at O is,

$$\sigma_{v_O} = h_1\gamma_w + h_2\gamma_{sat}$$

If γ_w is the unit weight of water then pore pressure u_O at O is,

$$u_o = (h_1 + h_2 + x)\gamma_w$$

If γ_{sat} and γ' saturated and submerged unit weights of the soil column respectively,

Then effective stress at O is,

$$\sigma' = \sigma_{v_O} - u_o = (h_1\gamma_w + h_2\gamma_{sat}) - (h_1 + h_2 + x)\gamma_w$$

$$\sigma' = h_2(\gamma_{sat} - \gamma_w) - x\gamma_w = h_2\gamma' - x\gamma_w$$

For quick sand condition(sand boiling) the effective stress tends to zero;

that is, $\sigma' = 0$

We get critical hydraulic gradient $i_{critical}$ as,

$$\sigma' = h_2\gamma' - x\gamma_w = 0;$$

$$i_{critical} = \frac{x}{h_2} = \frac{\gamma'}{\gamma_w} = \frac{\gamma_w(G-1)}{1+e} \times \frac{1}{\gamma_w} = \frac{G-1}{1+e}$$

Where G is the specific gravity of the soil particles and e is the void ratio of the soil mass. Therefore critical hydraulic gradient corresponds to hydraulic gradient which tends to a state of zero effective stress. Hence critical hydraulic gradient is given by

$$i_{critical} = \frac{G-1}{1+e}$$

4.12 Capillary water in soil

- Capillary rise results from the combined actions of surface tension and inter-molecular forces between the liquid and solids.
- The rise of water in soils above the ground water table is analogous to the rise of water into capillary tubes placed in a source of water.
- But, the void spaces in a soil are irregular in shape and size, as they interconnect in all directions.
- The pressure on the water table level is zero, any water above this level must have a negative pressure.
- In soils a negative pore pressure increases the effective stresses and varies with the degree of saturation

Capillary rise in glass tube:

Consider capillary tube of diameter d, capillary rise, h_c in the tube can be computed equating the surface tension forces to weight of water column that is raised due to capillary action as follows, (Refer to Figure 4.10)

$$(\pi d)\times T_s \cos\alpha = \frac{\pi d^2}{4} h_c \times \gamma_w$$

$h_c = \dfrac{4T_s \cos\alpha}{d\gamma_w}$; For pure water and a glass tube of very small diameter, $\alpha = 0$, $\therefore h_c = \dfrac{4T_s}{d\gamma_w}$

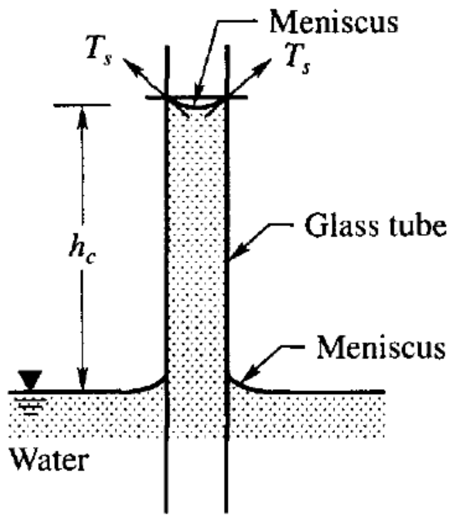


Figure 4.10: Capillary rise due to surface tension

The maximum negative pore pressure is: $u = h_c\gamma_w = \dfrac{4T_s}{d}$

The surface tension, T_S for water at 20° C is equal to 75 x 10^{-8} kN/cm.

- The capillary process starts as water evaporates from the surface of the soil.
- The capillary zone is comprised of a fully saturated layer with a height of usually less than h_c, and a partially saturated layer overlain by wet or dry soil.
- Negative pore pressure results in an increase in the effective stress and is termed soil suction.
- In granular materials (gravels and sands) the amount of capillary rise is negligible while in fine grained soils the water may rise up to several meters.

EXAMPLE: 4.1

The results of a constant head permeability test on fine sand are as follows: area of the soil specimen 180 cm^2, length of specimen 320 mm, constant head maintained 460 mm, and flow of water through the specimen 200mL in 5 min. Determine the coefficient of permeability.

Data: L = 320 mm = 32 cm, Q = 200 ml.=200 cm^3, A =180 cm^2, t = 5 minutes = 300 s
h = 460 mm = 46 cm

$$\therefore \boldsymbol{k} = \frac{\boldsymbol{Q}}{\boldsymbol{At}}\frac{\boldsymbol{L}}{\boldsymbol{h}} = \frac{200}{180\times 300}\frac{32}{46} = 0.00258\ cm/s$$

$$\therefore \boldsymbol{k} = 0.0258\ mm/s$$

EXAMPLE: 4.2

The discharge of water collected from a constant head permeameter in a period of 15 minutes is 400 ml. The internal diameter of the permeameter is 6 cm and the measured difference in heads between the two gauging points 15 cm apart is 40. Calculate the coefficient of permeability. If the dry weight of the 15 cm long sample is 7.0 N and the specific gravity of the solids is 2.65, calculate the seepage velocity.

Data – I:
L = 15 cm
$A = \pi D^2/4 = \pi(36)/4 = 28.27\ cm^2$
h = 40 cm
Q = 400 ml.=400 cm^3
t = 15 minutes = 900 s

Data – II:
G = 2.65
L=15 cm
$W_{dry} = 7.0$ N

$$\therefore \boldsymbol{k} = \frac{\boldsymbol{Q}}{\boldsymbol{At}}\frac{\boldsymbol{L}}{\boldsymbol{h}} = \frac{400}{28.27\times 900}\frac{15}{40} = 0.0059\ cm/s$$

Volume of the sample is

$$= AL = 28.27\times 15 = 424.05\ cm^3$$

$$Dry\ density\ \gamma_{dry} = \frac{7}{424.05}\times 1000 = 16.51\frac{kN}{m^3}$$

$$\gamma_{dry} = \frac{G\gamma_w}{1+e};\ \therefore 1+e = \frac{G\gamma_w}{\gamma_{dry}} = \frac{2.65\times 9.81}{16.51} = 1.575;\ \therefore e = 0.575$$

$$n = \frac{e}{1+e} = \frac{0.575}{1.575} = 0.365;\ \text{Superficial velocity,}$$

$$v = \frac{Q}{At} = \frac{0.0059}{28.27\times 900} = 0.016\ cm/s$$

$$\therefore\ seepage\ velocity,\ v_s = \frac{v}{n} = \frac{0.016}{0.365} = 0.0438\ cm/s$$

EXAMPLE: 4.3

In a falling head test permeability test initial head of 1.0 m dropped to 0.35 m in 3 hours, the diameter being 5mm. The soil specimen is 200 mm long and 100 mm in diameter. Calculate coefficient of permeability of the soil.

Data:
L = 200 mm
$A = \pi D^2/4 = \pi(100^2)/4 = 7853.98\ mm^2$; $a = \pi D^2/4 = \pi(5^2)/4 = 19.635\ mm^2$
t = 3 hours=180 minutes = 10800 s
h_1 = 1 m = 1000 mm, h_2 = 0.35 m = 350 mm

$$k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$

$$k = 2.303\frac{19.635\times 200}{7853.98\times 10800}\log_{10}\left(\frac{1000}{350}\right) = 4.86\times 10^{-5} mm$$

EXAMPLE: 4.4

A falling head permeability test is to be performed on a soil sample whose permeability is estimated to be about 3 × 10^{-5} cm/s. What diameter of the standpipe should be used if the head is to drop from 27.5 cm to 20.0 cm in 5 minutes and if the cross-sectional area and length of the sample are respectively 15 cm^2 and 8.5 cm ? Will it take the same time for the head to drop from 37.7 cm to 30.0 cm ?

Data – I: $L = 8.5$ cm $A = 15$ cm^2;
$t = 5$ minutes = 300 s $h_1 = 27.5$ cm, $h_2 = 20.0$ cm
Data – II: $h_1 = 37.7$ cm, $h_2 = 30.0$ cm

$$k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right);\Rightarrow 3\times10^{-5} = 2.303\frac{a\times8.5}{15\times300}\log_{10}\left(\frac{27.5}{20}\right)$$

$$\therefore\ a = \frac{3\times10^{-5}\times15\times300}{2.303\times8.5\times\log_{10}\left(\frac{27.5}{20}\right)} = 0.049864\ cm^2;$$

$$\Rightarrow d = \sqrt{\frac{4\times0.049864}{\pi}}\times10 = 2.52\ mm$$

Time required for $h_1 = 37.7\ cm$ to $h_2 = 30.0\ cm$

$$t = 2.303\frac{aL}{Ak}\log_{10}\left(\frac{h_1}{h_2}\right);\Rightarrow t = 2.303\frac{0.049864\times8.5}{15\times3\times10^{-5}}\log_{10}\left(\frac{37.7}{30}\right)$$

$$t = 215.22\ s < t = 300\ s\ \therefore it\ requires\ less\ time$$

EXAMPLE: 4.5

A sand deposit of 12 m thick overlies a clay layer. The water table is 3 m below the ground surface. In a field permeability pump-out test, the water is pumped out at a rate of 540 liters per minute when steady state conditions are reached. Two observation wells are located at 18 m and 36 m from the centre of the test well. The depths of the drawdown curve are 1.8 m and 1.5 m respectively for these two wells. Determine the coefficient of permeability.

Data:
$r_1 = 18\ m$, $r_2 = 36\ m$
$h = 12 – 3 = 9.0$ m
$d_1 = 1.8\ m$, $d_2 = 1.5\ m$
$\Rightarrow z_1 = 9.0 – 1.8 = 7.2\ m\ \&\ z_2 = 9.0 – 1.5 = 7.5\ m$
$q = 540\ lit / min = 540 /(60\times1000) = 0.009\ m^3/s$

(Refer to Figure E-4.5)

$$k = \frac{2.303q}{\pi(z_2^2 - z_1^2)}\left[\log_{10}\frac{r_2}{r_1}\right] = \frac{2.303\times0.009}{\pi(7.5^2 - 7.2^2)}\left[\log_{10}\frac{36}{18}\right] = 4.504\times10^{-4}\ m/s$$

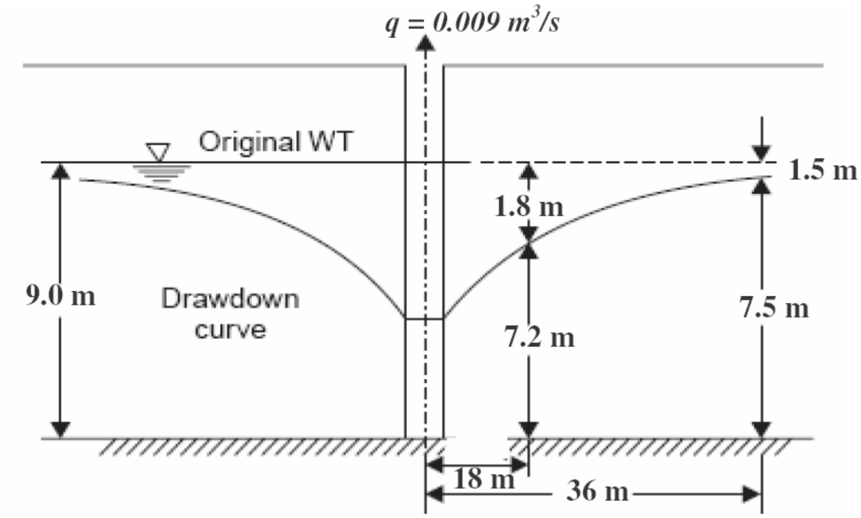


Figure E-4.5

EXAMPLE: 4.6

A pumping test carried out in a 50 m thick confined aquifer results in a flow rate of 600 lit/min. Drawdown in two observation wells located 50 m and 100 an from the well are 3 and 1 m respectively. Calculate the coefficient of permeability of the aquifer

Data:
$r_1 = 50\ m,\ r_2 = 100\ m$
$H = 50\ m$
$d_1 = 3.0\ m,\ d_2 = 1.0\ m$
$\Rightarrow\ z^1 = 70.0 - 3.0 = 67.0\ m\ \&\ z^2 = 70.0 - 1.0 = 69.0\ m$
$q = 600\ lit/min = 600/(60\times1000) = 0.010\ m^3/s$
(Refer to Figure E-4.6)

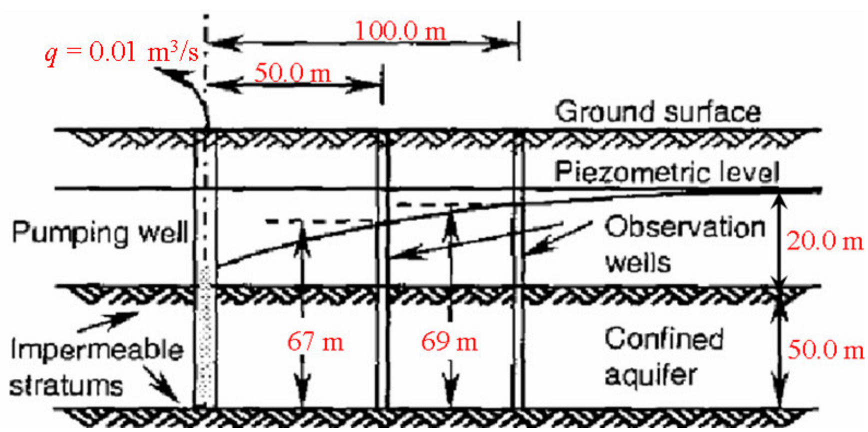


Figure E-4.6

$$k = \frac{q}{2.7283\times H(z_2 - z_1)}\left(\log_{10}\frac{r_2}{r_1}\right) = \frac{0.01}{2.7283\times50(69-67)}\left(\log_{10}\frac{100}{50}\right)$$

$$k = 1.1034\times10^{-5}\ m/s$$

EXAMPLE: 4.7

A pump test was carried out in an unconfined aquifer of k = 3 x 10-6 m/s with a flow rate of 20 m3/hour. The radius of the well is 0.4 m and the aquifer has a depth of 80 m above an impermeable stratum. The drawdown in an observation well at a distance of 150 m from the well is 2.5 m. Calculate the radius of influence and the depth of water in the well.

Data:
$r_1 = rw = 0.4\ m,\ r_2 = 150\ m,\ R = ?$
$h = 80.0\ m$
$d_2 = 2.5\ m,\ d_1 = d_w = ?$
$\Rightarrow\ z_2 = 80.0 - 2.5 = 77.5\ m,\ z_1 = z_w?$
$q = 20.0\ m^3/hour = \frac{1}{180}\ m^3/s,\ k = 3 \times 10^{-6}\ m/s$

$$k = \frac{2.303q}{\pi(z_2^2 - z_w^2)}\left[\log_{10}\frac{r_2}{r_w}\right] = \frac{2.303\times\left(\frac{1}{180}\right)}{\pi(77.5^2 - z_w^2)}\left[\log_{10}\frac{150}{0.40}\right] = 3.0\times10^{-6}\ m/s$$

$$z_w^2 = \left(77.5^2\right) - \left[\frac{2.303\times\log_{10}(375.0)}{180\times3.0\times10^{-6}\times\pi}\right] \Rightarrow z_w = 50.12\ m \Rightarrow d_w = 80 - 50.12 = 29.88\ m$$

$$k = \frac{q}{\pi(h - z_w^2)}\left[\log_e\frac{R}{r_w}\right] = \frac{\left(\frac{1}{180}\right)}{\pi(80^2 - 50.12^2)}\left[\log_e\frac{R}{0.40}\right] = 3.0\times10^{-6}\ m/s$$

$$\left[\log_e\frac{R}{0.40}\right] = 3.0\times10^{-6}\left[\pi(80^2 - 50.12^2)\times180\right]$$

$$\Rightarrow R = 0.40\times\exp\left[3.0\times10^{-6}\times\pi(80^2 - 50.12^2)\times180\right] \Rightarrow R = 292.81\ m$$

EXAMPLE: 4.8

The following data relate to a pump-out test:
Diameter of well = 24 *cm* → (*rw*)
Thickness of confined aquifer = 27 *m* → (*H*)
Radius of circle of influence = 333 *m* → (*R*)
Draw down during the test = 4.5 *m* → (zw = *h - 4.5*)
Discharge = 0.9 *m3/s*. What is the permeability of the aquifer ?

$$k = \frac{q}{2.7283\times H(z_2 - z_1)}\left(\log_{10}\frac{r_2}{r_1}\right)\quad for\ z_1 = h - d_1;\ z_2 = h - d_2$$

$$\Rightarrow k = \frac{q}{2.7283\times H(d_2 - d_1)}\left(\log_{10}\frac{r_2}{r_1}\right)$$

$$k = \frac{q}{2.7283 \times H(d_w - 0)}\left(\log_{10}\frac{R}{r_w}\right) = \frac{0.9}{2.7283 \times 27 \times (4.5-0)}\left(\log_{10}\frac{333}{0.12}\right)$$

$$k = 9.35 \times 10^{-3}\ m/s$$

EXAMPLE: 4.9

A soil profile consists of three layers with the properties shown in the table below. Calculate the equivalent coefficients of permeability parallel and normal to the stratum.

Layer	Thickness (m)	*k (m/s)*
1	3.0	2.0x 10^{-6}
2	4.0	5.0x 10^{-8}
3	3.0	3.0x 10^{-5}

Parallel to the layers.

$$k_h = \left[\frac{k_1H_1 + k_2H_2 + k_3H_3}{(H_1 + H_2 + H_3)}\right] = \left[\frac{(2\times10^{-6}\times3) + (5\times10^{-8}\times4) + (3\times10^{-5}\times3)}{(3+4+3)}\right] = 9.62\times10^{-6}\ m/s$$

Normal to the layers.

$$k_v = \frac{H}{\left(\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}\right)} = \left[\frac{10}{\left(\frac{3}{2\times10^{-6}}\right) + \left(\frac{4}{5\times10^{-8}}\right) + \left(\frac{3}{3\times10^{-5}}\right)}\right] = 1.23\times10^{-7}\ m/s$$

EXAMPLE: 4.10

The data given below relate to two falling head permeameter tests performed on two different soil samples:

(a) stand pipe area = 4 cm^2, (b) sample area = 28 cm^2,
(c) sample height = 5 cm, (d) initial head in the stand pipe =100 cm,
(e) final head = 20 cm,
(f) time required for the fall of water level in test 1, t = 500 sec,
(g) for test 2, t = 15 sec.

Determine the values of k for each of the samples. If these two types of soils form adjacent layers in a natural state with flow (a) in the horizontal direction, and (b) flow in the vertical direction, determine the equivalent permeability for both the cases by assuming that the thickness of each layer is equal to 150 cm.

Test − 1 :

$$\Rightarrow k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right) = \frac{2.303\times4\times5}{28\times500}\log\frac{100}{20} = 2.3\times10^{-3}\ cm/s$$

Test − 2:

$$\Rightarrow k = 2.303\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right) = \frac{2.303\times4\times5}{28\times15}\log\frac{100}{20} = 76.65\times10^{-3}\ cm/s$$

Horizontal direction flow

$$k_h = \left[\frac{k_1H_1 + k_2H_2}{(H_1 + H_2)}\right] = \left[\frac{(2.3\times10^{-3}\times150) + (76.65\times10^{-3}\times150)}{(150+150)}\right] = 0.0395 \ \ cm/s$$

Vertical direction flow

$$k_v = \frac{H}{\left(\frac{H_1}{k_1} + \frac{H_2}{k_2}\right)} = \left[\frac{300}{\left(\frac{150}{2.3\times10^{-3}}\right) + \left(\frac{150}{5\times10^{-3}}\right)}\right] = 4.466\times10^{-3} \ \ cm/s$$

EXAMPLE: 4.11

A layer of clay of 4 m thick is overlain by a sand layer of 5 m, the top of which is the ground surface. The clay overlay an impermeable stratum. Initially the water table is at the ground surface but it is lowered 4 meters by pumping. Calculate σ'_v at the top and base of the clay layer before and after pumping. For sand e = 0.45, G = 2.6, S_r (sand, after pumping) = 50%. For clay e = 1.0, G = 2.7.

$$(\gamma_b)_{sand\,(S_r=50\%)} = \frac{G\gamma_w(1+w)}{1+e} = \frac{2.6\times9.81\times(1+0.08654)}{1+0.45} = 19.113 \ \ kN/m^3$$

$$w = \frac{eS_r}{G} = \frac{0.45\times.50}{2.7} = 0.08654$$

$$(\gamma_{sat})_{sand} = \frac{\gamma_w(G+e)}{1+e} = \frac{9.81\times(2.6+0.45)}{1+0.45} = 20.635 \ \ kN/m^3$$

$$(\gamma_{sat})_{clay} = \frac{\gamma_w(G+e)}{1+e} = \frac{9.81\times(2.7+1.0)}{(1+1)} = 18.15 \ \ kN/m^3$$

At the top of the clay layer before pumping:

σ_v = 20.635×5.0 = 103.175 *kPa,*

u = 9.81×5.0 = 49.0 *kPa,*

σ_v' = 103.175 - 49.0 = 54.175 *kPa.*

At the base of the clay layer before pumping:

σ_v = 103.175 + 18.15x4.0 = 175.775 *kPa.*

u = 9.81×9.0= 88.3 *kPa,*

σ_v' = 175.775 - 88.3 = 87.475 *kPa*

At the top of the clay layer after pumping:

σ_v = 19.113×4.0+ 20.635×1.0 = 97.087 *kPa,*

u = 9.81×1.0 = 9.81 *kPa,*

σ_v' = 97.087 - 9.81 = 87.277 *kPa.*

At the base of the clay layer after pumping:

σ_v = 97.087 + 18.15×4.0 = 169.687 *kPa,*

u = 9.81×5.0 = 49.05 *kPa*,

σ_v' = 172.99 -49.05 = 120.637 *kPa.*

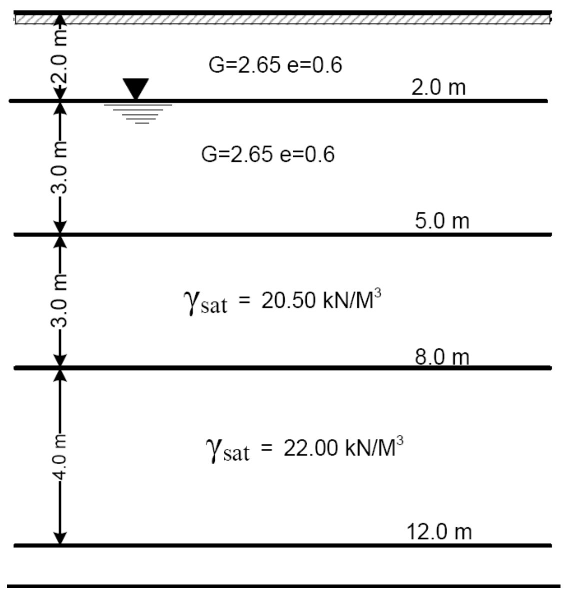
$\Delta\sigma_v$' (at the top) = 87.277 – 54.175 = 33.102 *kPa.*

$\Delta\sigma_v$' (at the base) = 120.637 – 87.475 = 33.162 *kPa*.

The increase in the effective vertical stress throughout the clay layer is uniform.

EXAMPLE: 4.12

A soil profile is shown in figure. Plot the distribution of total stress, pore pressure and effective stress up to a depth of 12 m.



$\underline{0.0\ m - 2.0\ m}$

$$\gamma_{dry} = \frac{G\gamma_w}{(1+e)} = \frac{2.65\times 9.80}{(1+0.6)} = 16.23\ kN/m^3$$

$\underline{2.0\ m - 5.0\ m}$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{(1+e)} = \frac{(2.65+0.6)\times 9.80}{(1+0.6)} = 19.91\ kN/m^3$$

$\underline{5.0\ m - 8.0\ m}$

$$\gamma_{sat} = 20.5\ kN/m^3$$

$\underline{8.0\ m - 12.0\ m}$

$$\gamma_{sat} = 22.0\ kN/m^3$$

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$\underline{z = 0.0\ \ m} \Rightarrow u = 0.0,\ \ \sigma = 0.0,\ \ \sigma' = 0.0$

$\underline{z = 2.0\ \ m} \Rightarrow u = 0.0,\ \ \sigma = 16.23 \times 2 = 32.46\ \ kN / m^3,$

$\sigma' = 32.46 - 0.0 = 32.46\ \ kN / m^3$

$\underline{z = 5.0\ \ m} \Rightarrow u = 3 \times 9.8 = 29.4\ \ kN / m^3,\ \ \sigma = 32.46 + 19.91 \times 3 = 92.19\ \ kN / m^3,$

$\sigma' = 92.19 - 29.4 = 62.79\ \ kN / m^3$

$\underline{z = 8.0\ \ m} \Rightarrow u = 6 \times 9.8 = 58.80\ \ kN / m^3,\ \ \sigma = 92.19 + 20.50 \times 3 = 153.69\ \ kN / m^3,$

$\sigma' = 153.69 - 58.80 = 94.89\ \ kN / m^3$

$\underline{z = 12.0\ \ m} \Rightarrow u = 10 \times 9.8 = 98.0\ \ kN / m^3,$

$\sigma = 153.69 + 22.0 \times 4 = 241.69\ \ kN / m^3,$

$\sigma' = 241.69 - 98.0 = 143.69\ \ kN / m^3$

Depth (m)	***$u = z\ \gamma_w$ (kPa)***	***$\sigma = z\ \gamma_{sat}$ (kPa)***	***$\sigma' = \sigma - u$ (kPa)***
0.0	0.00	0.00	0.00
2.0	0.00	32.46	32.46
5.0	29.40	92.19	62.79
8.0	58.80	153.69	94.89
12.0	98.00	241.69	94.89

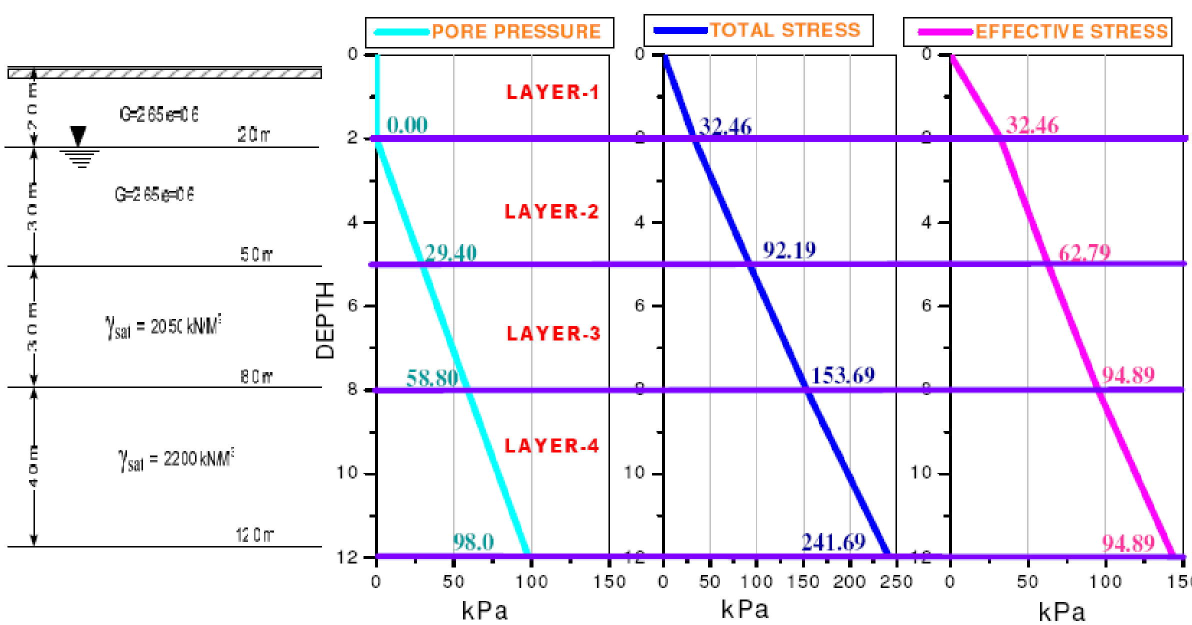

Figure: Example: 4.12 - Pore pressure, Total stress and Effective stress Distribution

EXAMPLE: 4.13

If a glass tube of 0.002 mm diameter is immersed in water, what is the height to which water will rise in the tube by capillary action? Derive the necessary expression for capillary rise and use the same.

T_S = 75 x 10-8 kN/cm. = 75 x 10-6 kN/m.
D = 0.002 × 10^{-3} m
γ_w = 9.80 kN/m^3

$$h_c = \frac{4T_s}{d\gamma_w} = \frac{4\times75\times10^{-6}}{0.002\times10^{-3}\times9.80}$$

$$h_c = 15.36\ m$$

EXAMPLE: 4.14

What is the height of capillary rise in a soil with an effective size of 0.06 mm and void ratio of 0.72 ?

$$Effective\ size = 0.05\ mm$$

$$Volume\ of\ solids = (0.05)^3\ mm^3$$

$$voidratio = 0.72$$

$$Volume\ of\ voids = 0.72\times(0.05)^3 = 9\times10^{-5}\ mm^3$$

$$Approximately, void\ size,\ d = (9\times10^{-5})^{\frac{1}{3}} = 0.0448\ mm$$

$$\text{Capillary rise, } h_c = \frac{4T_s}{d\gamma_w} = \frac{4\times75\times10^{-6}}{0.0448\times9.80\times10^{-3}} = 0.683\ m$$

EXAMPLE: 4.15

Water is flowing at the rate of 50 mm^3/s in an upward direction through a sample of sand whose coefficient of permeability is 2x10 $^{-2}$ mm/s. The sample thickness is 120 mm and cross section area is 5000 mm^2. Determine the effective pressure at the middle and at bottom sections of the sample. Take the saturated unit weight of sand as 19 KN/m^3.

$$q = 50\ mm^3/s;\ k = 2\times10^{-2}\ mm/s;\ A = 5000\ mm^2;\ z = 120\ mm$$

$$\gamma' = \gamma_{sat} - \gamma_w = 19.0 - 9.80 = 9.2\ kN/m^3$$

$$\text{Hydraulic gradient} = i = \frac{q}{kA} = \frac{50}{2\times10^{-2}\times5000} = 0.5$$

$$\text{For upward flow,} \Rightarrow \sigma' = z\times(\gamma' - i\gamma_w)$$

$$\text{At the bottom; } z = 120\ mm; \Rightarrow \sigma' = 120\times10^{-3}\times(9.2 - 0.5\times9.8)$$

$$(\sigma')_{z=120mm} = 0.516\ kPa$$

$$\text{At the middle; } z = 60\ mm; \Rightarrow \sigma' = 60\times10^{-3}\times(9.2 - 0.5\times9.8)$$

$$(\sigma')_{z=60mm} = 0.258\ kPa$$

EXAMPLE: 4.16

A 1.60 m layer of the soil of specific gravity, G = 2.66 and porosity, n= 38% is subject to an upward seepage head of 2.0 m. What depth of coarse sand would be required above the soil to provide a factor of safety of 1.5 against quick sand condition assuming that the coarse sand has the same porosity and specific gravity as the soil and that there is negligible head loss in the sand?

$$G = 2.66;\ n = 38\% \Rightarrow e = \frac{n}{1-n} = \frac{0.38}{1-0.38} = 0.613$$

$$Critical\ hydraulic\ gradient; \Rightarrow i_c = \frac{G-1}{1+e} = \frac{2.66-1}{1+0.613} = 1.029$$

$$Length\ of\ flow\ required\ is,\ L = \frac{h}{i} = \frac{2}{0.686} = 2.916\ m$$

$$Available\ thickness\ of\ soil = 1.60\ m$$

$$\therefore\ Additional\ thickness\ of\ sand\ layer\ required$$

$$d = 2.916 - 1.60 = 1.316\ m$$

$$Additional\ thickness\ of\ sand\ layer\ required = 1.316\ m$$

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Reference:

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