Lesson 3

Matrix relation of DFT
The DFT expression can be expressed as

\[ [X] = [x(n)] [WN]^T \]

where \([X] = [X(0), X(1), \ldots] \)

[x] is the transpose of the input sequence. WN is a N x N matrix

\[ WN = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & wn1 & wn2 & wn3 & \cdots & wn \ n-1 \\ 1 & wn2 & wn4 & wn6 & \cdots & wn(n-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1 \end{bmatrix} \]

ex;
4 pt DFT of the sequence 0, 1, 2, 3

\[
\begin{align*}
X(0) &= 1 \quad 1 \quad 1 \quad 1 \\
X(1) &= 1 \quad -j \quad -1 \quad j \\
X(2) &= 1 \quad -1 \quad 1 \quad -1 \\
X(3) &= 1 \quad j \quad -1 \quad -j
\end{align*}
\]

Solving the matrix \(X(K) = 6, -2+2j, -2, -2-2j\)

Properties of DFT:

Linearity:

\[ a x_1(n) + b x_2(n) \longleftrightarrow a X_1(k) + b X_2(k) \]
**Circular shift:**

In linear shift, when a sequence is shifted the sequence gets extended. In circular shift the number of elements in a sequence remains the same. Given a sequence $x(n)$ the shifted version $x(n-m)$ indicates a shift of $m$. With dfts the sequences are defined for 0 to $N-1$.

If $x(n) = x(0), x(1), x(2), x(3)$

$X(n-1) = x(3), x(0), x(1), x(2)$

$X(n-2) = x(2), x(3), x(0), x(1)$
Lesson 4

Time shift thm:

If \( x(n) \leftrightarrow X(k) \)

Then \( x(n-m) \leftrightarrow W_N X(k) \)

Frequency shift

If \( x(n) \leftrightarrow X(k) \)

\[ +nok \]

\[ W_n \quad x(n) \leftrightarrow X(k+no) \]

\[ N-1 \quad kn \]

Consider \( x(k) = \sum_{n=0}^{N-1} x(n) W_n \)

\[ (k+no)n \]

\[ X(k+no)=\sum_{n=0}^{N-1} x(n) W_N \]

\[ kn \quad non \]

\[ = \sum x(n) W_N \]

\[ WN \quad non \]

\[ \therefore X(k+no) \leftrightarrow x(n) W_N \]
Lesson 5

Symmetry:

For a real sequence, if $x(n) \leftrightarrow X(k)$

$$X(N-K) = X^* (k)$$

For a complex sequence

$$\text{DFT}(x^*(n)) = X^*(N-K)$$

If $x(n)$ then $X(k)$

| Real and even       | real and even |
| Real and odd        | imaginary and odd |
| Odd and imaginary   | real odd |
| Even and imaginary  | imaginary and even |

Convolution theorem:

Circular convolution in time domain corresponds to multiplication of the DFTs

If $y(n) = x(n) \otimes h(n)$ then $Y(k) = X(k) H(k)$

Ex let $x(n) = 1,2,2,1$ and $h(n) = 1,2,2,1$
Then $y(n) = x(n) \otimes h(n)$

$Y(n) = 9,10,9,8$
Lesson 6

2. N pt DFTs of 2 real sequences can be found using a single DFT

If \( g(n) \) & \( h(n) \) are two sequences then let \( x(n) = g(n) + jh(n) \)

\[
G(k) = \frac{1}{2} (X(k) + X^*(k)) \\
H(k) = \frac{1}{2}j (X(K) + X^*(k))
\]

2N pt DFT of a real sequence using a single N pt DFT

let \( x(n) \) be a real sequence of length 2N with \( y(n) \) and \( g(n) \) denoting its N pt dft

let \( y(n) = x(2n) \) and \( g(2n+1) \)

\[
X(k) = Y(k) + WN \ G(k)
\]

Using DFT to find IDFT

The DFT expression can be used to find IDFT

\[
X(n) = \frac{1}{N} [DFT(X^*(k))]^*
\]
Lesson 7
Digital filtering using DFT

In a LTI system the system response is got by convoluting the input with the impulse response. In the frequency domain their respective spectra are multiplied. These spectra are continuous and hence cannot be used for computations. The product of 2 DFT’s is equivalent to the circular convolution of the corresponding time domain sequences. Circular convolution cannot be used to determine the output of a linear filter to a given input sequence. In this case a frequency domain methodology equivalent to linear convolution is required. Linear convolution can be implemented using circular convolution by taking the length of the convolution as $N \geq n_1 + n_2 - 1$ where $n_1$ and $n_2$ are the lengths of the 2 sequences.

Overlap and add

In order to convolve a short duration sequence with a long duration sequence $x(n), x(n)$ is split into blocks of length $N$ $x(n)$ and $h(n)$ are zero padded to length $N + K - 1$. Circular convolution is performed to each block then the results are added.

Overlap and save method

In this method $x(n)$ is divided into blocks of length $N$ with an overlap of $k-1$ samples. The first block is zero padded with $k-1$ zeros at the beginning. $H(n)$ is also zero padded to length $N$. Circular convolution of each block is performed using the $N$ length DFT. The output signal is obtained after discarding the first $k-1$ samples. The final result is obtained by adding the intermediate results.